International Journal of Heat and Mass Transfer 52 (2009) 3187–3196

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Review of convective heat transfer enhancement with nanofluids

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A R T I C L E I N F O

Article history:
Received 6 January 2009
Received in revised form 2 February 2009
Accepted 2 February 2009
Available online 26 March 2009

Keywords:
Nanofluids
Effective thermal conductivity
Heat transfer enhancement

A B S T R A C T

Nanofluids are considered to offer important advantages over conventional heat transfer fluids. Over a decade ago, researchers focused on measuring and modeling the effective thermal conductivity and viscosity of nanofluids. Recently important theoretical and experimental research works on convective heat transfer appeared in the open literatures on the enhancement of heat transfer using suspensions of nanometer-sized solid particle materials, metallic or nonmetallic in base heat transfer fluids. The purpose of this review article is to summarize the important published articles on the enhancement of the forced convection heat transfer with nanofluids.

1. Introduction

Nanofluid is envisioned to describe a fluid in which nanometer-sized particles are suspended in conventional heat transfer basic fluids. Conventional heat transfer fluids, including oil, water, and ethylene glycol mixture are poor heat transfer fluids, since the thermal conductivity of these fluids play important role on the heat transfer coefficient between the heat transfer medium and the heat transfer surface. Therefore numerous methods have been taken to improve the thermal conductivity of these fluids by suspending nano/micro or larger-sized particle materials in liquids.

Since the solid nanoparticles with typical length scales of 1–100 nm with high thermal conductivity are suspended in the base fluid (low thermal conductivity), have been shown to enhance effective thermal conductivity and the convective heat transfer coefficient of the base fluid. The thermal conductivity of the particle materials, metallic or nonmetallic such as Al_2O_3, CuO, Cu, SiO, TiO, are typically order-of-magnitude higher than the base fluids even at low concentrations, result in significant increases in the heat transfer coefficient (Table 1). Therefore the effective thermal conductivity of nanofluids is expected the enhanced heat transfer compared with conventional heat transfer liquids.

Choi [2] is the first who used the term nanofluids to refer to the fluid with suspended nanoparticles. Choi et al. [3] showed that the addition of a small amount (less than 1% by volume) of nanoparticles to conventional heat transfer liquids increased the thermal conductivity of the fluid up to approximately two times.

Several researches Masuda et al. [4], Lee et al. [5], Xuan and Li [6], and Xuan and Roetzel [7] stated that with low nanoparticles concentrations (1–5 Vol%), the thermal conductivity of the suspensions can increase more than 20%. Eastman et al. [1] at Argonne National laboratory showed with some preliminary experiments with suspended nanoparticles, the thermal conductivity of approximately 60% can be obtained with 5 Vol% CuO nanoparticles in the based fluid of water. Heat transfer coefficient is the determining factor in forced convection cooling–heating applications of heat exchange equipments including engines and engine systems. Such enhancement mainly depends upon factors such as particle volume concentration, particle material, particle size, particle shape, base fluid material temperature, and additives.

Nanoparticles used in nanofluids have been made out of many materials by physical and chemical synthesis processes. Typical physical methods include the mechanical grinding method and the inert-gas-condensation technique [8].

Current processes specifically for making metal nanoparticles include mechanical milling, inert-gas-condensation technique, chemical precipitation, chemical vapor deposition, micro-emulsions, spray pyrolysis and thermal spraying. Nanoparticles in most materials discussed are most commonly produced in the form of powders [9]. In powder form, nanoparticles can be dispersed in aqueous or organic base liquids to form nanofluids for specific applications. Up to date, nanofluids of various qualities have been produced mainly by small volumes by two-step technique and the single step technique which simultaneously produce powders and disperses directly into the base fluids [9]. The large-scale

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doi:10.1016/j.ijheatmasstransfer.2009.02.006

Nomenclature

a	acceleration
c_p	heat capacity
d	channel diameter
D	thermal dispersion coefficient
D_r	thermal dispersion coefficient in radial direction
D_x	thermal dispersion coefficient in axial direction
$\bar{D}$	effective apparent thermal diffusivity
g	gravity
h	convection coefficient
J_0	Bessel function of order zero of the first kind
K	thermal conductivity
$\bar{K}$	apparent thermal conductivity
n	empirical shape factor
Nu	Nusselt number
Nu_T	Nusselt number under constant wall temperature boundary condition
P	pressure
Pe	Peclet number
Pr	Prandtl number
r	distance in radial direction from the center of the channel
r_0	radius of the channel
Re	Reynolds number
T	temperature
T_b	fluid temperature
T_i	inlet temperature
T_m	mean temperature
T_w	wall temperature
u	velocity
$\bar{u}$	average velocity
U_m	slug flow velocity
$\bar{v}$	velocity vector
x	distance in axial direction
z	axial variable

Greek letters

α	thermal diffusivity
β	ratio between the nanolayer thickness and the original particle radius
λ	Eigen value
μ	viscosity
ρ	density
ϕ	particle volume fraction

Subscripts

bf	base fluid
d	the contribution of hydrodynamic dispersion and irregular movement of the nanoparticles
dr	drift condition
eff	effective
eq	equivalent
f	base fluid
m	mixture
n	nanomaterials
nf	nanofluid
p	solid particles
pf	slip condition
x	axial variable
1	liquid
2	particle

production of well-dispersed nanofluids at low cost is required for commercial applications [1].

2. Thermal conductivity of nanofluids

Since the high thermal conductivity nanoparticles suspended in the base fluid which has a low thermal conductivity, remarkably increase thermal conductivity of nanofluids. Researchers developed many models to tell how much that increase would be and many experiments have been conducted to compare experimental data with those analytical models. This still needs further research to develop a sophisticated theory to predict thermal conductivity of nanofluids. But there exists some empirical correlations to calculate effective thermal conductivity of two-phase mixture.

In the literature, the thermal conductivity enhancement ratio been defined as the ratio of thermal conductivity of the nanofluid to the thermal conductivity of the base fluid (K_{eff}/K_1). Researches developed their thermal conductivity models based on the classical research of Maxwell who researched conduction through heterogeneous media. The effective thermal conductivity for a two-phase mixture consisting of a continuous and discontinuous phase has been conducted by Maxwell [10] and the effective thermal conductivity K_{eff} is given by

$$K_{eff,Maxwell} = \frac{2K_2 + K_1 + \phi(K_2 - K_1)}{2K_2 + K_1 - 2\phi(K_2 - K_1)} K_1 \quad (1)$$

where K_1 and K_2 are the thermal conductivity of the liquid and the particle respectively and ϕ is the particle volume fraction. Maxwell derived his model based on the assumption that the discontinuous phase is spherical in shape and the thermal conductivity of nanofluids depend on the thermal conductivity of spherical particles, the base fluid and the particle volume fraction.

Hamilton and Crosser [11] extended Maxwell work to cover none spherical particles and introduced the shape factor (n) which can be determined experimentally for different type of materials. The goal of their research was to develop a model as a function of particle shape, composition and the conductivity of both continuous and discontinuous phases. Hamilton and Crosser model for a discontinuous phase (particles) dispersed in a continuous phase is:

$$K_{eff,Hamilton-Crosser} = K_1 \left[\frac{K_2 + (n-1)K_1 - (n-1)\phi(K_1 - K_2)}{K_2 + (n-1)K_1 + \phi(K_1 - K_2)} \right] \quad (2)$$

where the empirical shape factor (n) is defined by $n = 3/\Psi$ and Ψ is the sphericity defined as the ratio of the surface areas of a sphere with the volume equal to that of the particle. The Hamilton–Crosser model reduces to Maxwell model when $\Psi = 1$ and was found to be in agreement with experimental data for $\phi < 30\%$. The model is va-

Table 1
Thermal conductivities of various solids and liquids [1].

Solids/liquids	Material	Thermal conductivity (W/m K)
Metallic solids	Silver	429
	Copper	401
	Aluminum	237
Nonmetallic solids	Diamond	3300
	Carbon nanotubes	3000
	Silicon	148
	Alumina (Al_2O_3)	40
Metallic liquids	Sodium @ 644 K	72.3
Nonmetallic liquids	Water	0.613
	Ethylene glycol (EG)	0.253
	Engine oil (EO)	0.145

lid as long as the conductivity of the particles is larger than conductivity of the continuous phase by at least by a factor of 100. Although the experiments show that these models are good at predicting the thermal conductivity; the size effects of nanoparticles are not included in these models.

Yu and Choi [12] modified Maxwell model with the assumption that the base fluid molecules close to the solid surface of the nanoparticles form a solid-like layered structures. Hence the nanolayer works as a thermal bridge between the liquid base fluid and the solid nanoparticles, and this will enhance the effective thermal conductivity. As seen from Fig. 1, a nanofluid consists of the liquid base fluid, the solid nanoparticles and the nanolayers.

In order to include the effect of nanolayer in calculating K_{eff}, Yu and Choi assumed a spherical nanoparticle of radius (r) surrounded by a nanolayer of thickness (h), (Fig. 1). In addition, they assumed that the thermal conductivity of the nanolayer (K_{layer}) is higher than thermal conductivity of the liquid (K_1). When the nanolayer is combined with the nanoparticle, an "equivalent nanoparticle" with thermal conductivity of (K_{eq}) is introduced. The equivalent thermal conductivity can be calculated using the effective medium theory [13] as:

$$K_{eq} = \frac{[2(1-\gamma) + (1+\beta)^3(1+2\gamma)]\gamma}{-(1-\gamma) + (1+\beta)^3(1+2\gamma)} K_2 \tag{3}$$

where $\beta = h/r$ is the ratio between the nanolayer thickness and the original particle radius and $\gamma = K_{layer}/K_2$. For the case $\gamma = 1$, then $K_{layer} = K_2 = K_{eq}$

Hence, Yu and Choi modified Eq. (1) and produced the following model for the effective thermal conductivity:

$$K_{Yu-Choi} = \frac{K_2 + 2K_1 + 2\phi(K_2 - K_1)(1+\beta)^3}{K_2 + 2K_1 - \phi(K_2 - K_1)(1+\beta)^3} K_1 \tag{4}$$

It is important to note that the effective thermal conductivity of nanofluids depends on the thermal conductivity of solid particles and base fluid, particle volume fraction, shape of particles and the thickness and the thermal conductivity of nanolayer.

The comparison of these three important models can be made by assuming the nanolayer and radius of the nanoparticles. Yu and Choi [12] illustrated their thermal conductivity model by plotting the values for 1 nm and 2 nm-thick nanolayer cases. For the thermal conductivity of the nanolayer, a value between thermal conductivity of the base fluid and the thermal conductivity of nanoparticles is proper. 5 W/m K for thermal conductivity of the nanolayer with a thickness of 2 nm for the Al_2O_3/water nanofluid, and the Al_2O_3 nanoparticles were chosen as 15 nm for the comparison of the models. The results for these three models are shown in Fig. 2 for various values of the particle volume fraction ϕ. The figure shows that that all of the models predict increasing thermal conductivity ratio with increasing particle volume fraction. A linear relationship is present for all of the models. Highest values are obtained by using Hamilton–Crosser model. Models of Hamilton–Crosser and Yu and Choi are relatively comparable whereas Maxwell predicts much lower thermal conductivity ratios than these two models. The discrepancy between the models increases with increasing particle volume fraction.

An important review of experimental works on the effective thermal conductivity of nanofluids and heat transfer enhancement is given by Yu et al. [9]. Some of the following experimental findings are extracted from this work.

Lee et al. [5] and Wang et al. [14] studied the effect particle volume fraction with 24 and 23 nm CuO particles in a base fluid of water, and found that the thermal conductivity enhancement increases linearly with increased particle volume concentration; at 10% volume fraction, the thermal conductivity ratio increased by 34%.

The effect of particle volume concentration with ethylene glycol as base fluid with CuO nanoparticles is studied by Lee et al. [5] and Wang et al. [14]. They found that volume concentration of CuO particles with 15%, the thermal conductivity ratio increased by 50%. Several investigators agreed on the magnitude of enhancement with reportedly identical test parameters for Al_2O_3 in water [5,15–17]], CuO in water [5,14], and CuO in ethylene glycol [5,14].

Wang et al. [14], Lee et al. [5], Xie et al. [18–20], and Das et al. [15–17] carried out research by isolating the material property effect by keeping all the parameters such as particle size, base fluid, and temperature are approximately constant. But the situation changes with the particle material when higher conductivity particles such as Al_2O_3, CuO, SiC with the same particle size are used. The metal particles produce the same enhancement as the oxide particles but at much lower volume concentration. On the other hand, it is difficult to produce metal particle nanofluids without the particles oxidizing during the production process.

The effect of particle size is investigated by Wang et al. [14], Lee et al. [5], Xie et al. [18–20], and Das et al. [15–17] with spherical particles for a single particle–water combination over a range of particle diameter from 20 nm to 60 nm. General trend is that the large particle diameters produce a large enhancement in thermal conductivity. On the other hand, some theories predict that a uniform distribution of small particles produce better heat transfer enhancement.

Xie et al. [18] studied the effect of particle shape on the thermal conductivity enhancement in nanofluid and the results were compared with respect to the geometric shape of the particle with the same material and base fluid. The results indicate that elongated particles are better enhancement of the thermal conductivity. They

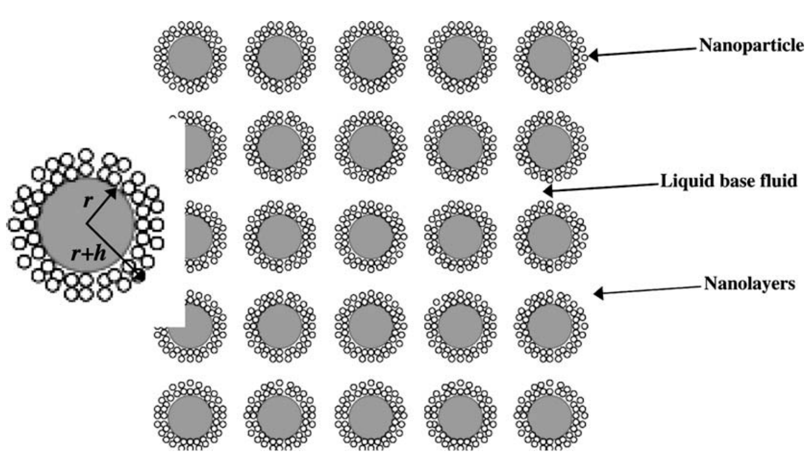

Fig. 1. Schematic cross section of Nanofluid structure [12].

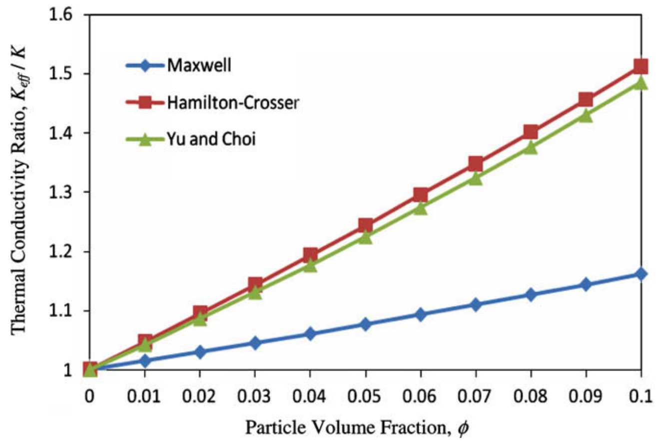

Fig. 2. Variation of thermal conductivity ratio with particle volume fraction for Al_2O_3/water nanofluid.

used 26 nm spherical and 600 nm cylindrical particles of SiC in ethylene glycol base fluid. It is found that at 3% volume concentration, the thermal conductivity ratio of 1.16 and 1.10 was obtained for cylindrical and spherical particles respectively.

Xie et al. [19] examined the effect of the base fluid material on the thermal conductivity enhancement. The results show increased thermal conductivity enhancement of the base fluid which has low thermal conductivity. These results are important for the design of the heat exchange equipment where heat transfer enhancement is needed.

The effect of temperature on the thermal conductivity enhancement has been studied by Das et al. [15], Wen and Ding [21,22], and Li and Peterson [23]. The data clearly indicated that the thermal conductivity enhancement increases with increased temperature.

To prevent nanoparticles from agglomerating fluid additives are used during the experimentation. In the literature, most studies involving additives such as thioglycolic acid show enhancement in the thermal conductivity [24].

3. Enhancement of convective heat transfer

The enhancement of the heat transfer coefficient is a better indicator than the thermal conductivity enhancement for nanofluids used in the design of heat exchange equipment. The physical properties of nanofluids are quite different than the base fluid. Density, specific heat and viscosity are also changed which enhance the heat transfer coefficient exceeding the thermal conductivity enhancement results reported by some experiments.

Heris et al. [25] did experiments with Al_2O_3 and CuO nanoparticles in water under laminar flow up to turbulence (Fig. 3). He found that more heat transfer enhancement as high as 40% with Al_2O_3 particles while the thermal conductivity enhancement was less than 15% [9].

Heat transfer experimental results are available from Pak and Cho [26], Xuan and Li [27], Yang et al. [28], Heris et al. [25], Ding et al. [29], Ma et al. [30], Chen et al. [31], and Kulkarni et al. [32].

Pak and Cho [26] performed experiments on turbulent heat transfer performance of two kinds of nanofluids and turbulent frictions by using γ-Al_2O_3 and TiO_2 dispersed in water. Xuan and Li [27] studied single-phase flow for the turbulent flow and developed the following heat transfer correlation for the experimental data:

$$Nu_{nf} = \frac{h_{nf} d}{K_{nf}} = 0.0059\left(1.0 + 7.6286\phi^{0.6886} Pe_d^{0.001}\right) \mathrm{Re}_{nf}^{0.9238} \mathrm{Pr}_{nf}^{0.4} \quad (5)$$

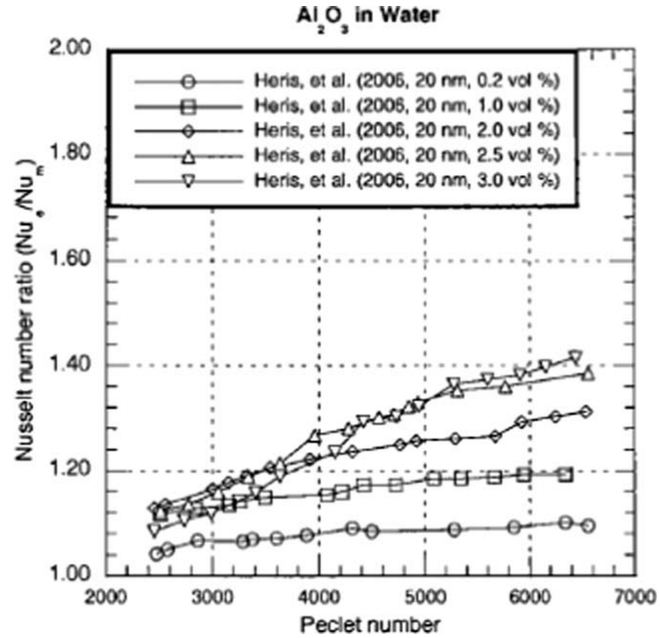

Fig. 3. Laminar flow heat transfer of Al_2O_3 in water [9].

For laminar flow Xuan and Li [27] provided also a correlation:

$$Nu_{nf} = \frac{h_{nf} d}{K_{nf}} = 0.4328\left(1.0 + 11.285\phi^{0.754} Pe_d^{0.218}\right) \mathrm{Re}_{nf}^{0.333} \mathrm{Pr}_{nf}^{0.4} \quad (6)$$

The Peclet number, *Pe*, describes the effect of thermal dispersion caused by microconvective and microdiffusion of the suspended particles. The particle Peclet number, Reynolds number and the Prandtl number for nanofluid are defined respectively as

$$Pe_d = \frac{u_m d_p}{\alpha_{nf}} \quad (7)$$

$$\mathrm{Re}_{nf} = \frac{u_m d}{\nu_{nf}} \quad (8)$$

and

$$\mathrm{Pr}_{nf} = \frac{\nu_{nf}}{\alpha_{nf}} \quad (9)$$

where the thermal diffusively is given by

$$\alpha_{nf} = \frac{K_{nf}}{(\rho c_p)_{nf}} = \frac{K_{nf}}{(1-\phi)(\rho c_p)_f + \phi(\rho c_p)_p} \quad (10)$$

4. Theoretical analysis of heat transfer enhancement with nanofluids

The seminal work by Choi [2] reported the concept of nanofluids and then the interest in this area has grown. Limited computer simulations of thermal properties and heat transfer characteristics of nanofluids have been performed. Some of these simulations dealt with the effective thermal conductivity of nanofluids [33,34] or effective viscosity [35] and most of them focuses on the heat transfer of nanofluids [36–60]. A complete understanding about the heat transfer enhancement in forced convection in laminar and turbulent flow with nanofluids is necessary for their practical applications to heat transfer enhancement. Nanofluids, in nature, are multi-component fluids. It is therefore in the literature available treated as either a two-phase homogeneous flow with no slip between nanoparticles and the fluid which are also in thermal equilibrium; or it is treated with a slip between the particles and the base fluid with thermal equilibrium.

Most forced convection flows are dependent on both the Reynolds and Prandtl numbers but for the case of nanofluids additional parameters are included to take into account the thermal properties of all the constituents. From the information mentioned above, it is expected that heat transfer coefficient of the nanofluid will depend on the thermal conductivity and the heat capacity of the base fluid and nanomaterials, flow pattern, Reynolds and Prandtl numbers, temperature, the volume fraction of the suspended particles, the dimensions, and shape of the particles.

Xuan and Roetzel [7] proposed the following general function for the Nusselt number:

$$Nu_{nf} = f\left[\mathrm{Re}, \mathrm{Pr}, \frac{K_n}{K_f}\frac{(\rho c_p)_n}{(\rho c_p)_f}, \phi, \textit{particle shape}, \textit{flow geometry}\right] \tag{11}$$

where f and n stand for base fluid and nanomaterials respectively. Another possible method of formulation suggested by Xuan and Roetzel [7] is by postulating that the ratio of the heat transfer coefficients of the nanofluid and base fluid is proportional to the ratio of the respective thermal conductivities of the nanofluid and base fluid raised to some power, m.

$$h_{nf} \approx h_f\left(\frac{K_{nf}}{K_f}\right)^m \tag{12}$$

where the exponent depends on flow regime and $m = 2/3$ is suggested for turbulent flow.

The above mentioned methods of formulation regard the nanofluid as a single phase fluid where in reality it is a two-phase solid–fluid mixture. The size of the dispersed particles presents some difficulty in analyzing the interaction between the fluid and the solid particles during energy transfer. Many researchers have suggested that in fact Brownian motion is one of the factors in the enhancement of heat transfer. This random motion of ultra-fine particles would create a slip velocity between the solid particles and the fluid medium. Xuan and Roetzel [7] also suggest including small perturbations in the temperature and velocity formulation to account for the Brownian motion.

Results showed significant increased with regards to both particle concentration and flow Reynolds number. Similarly an analysis was done with the ethylene glycol based mixture and results showed similar trends however the ethylene glycol mixture shows more pronounced variations with respect to the particle concentration and flow Reynolds number.

As mentioned above, there are two approaches in finding heat transfer coefficient of the nanofluids in duct flow. One of the methods is conventional approach by the use of the nanofluid thermal and transport properties in the available correlations for heat transfer coefficient for the pure base fluid. Therefore the following property expressions are for nanofluids

$$\rho_{eff} = (1-\phi)\rho_f + \phi\rho_n \tag{13}$$

$$c_{p,eff} = (1-\phi)c_{p,f} + \phi c_{p,n} \tag{14}$$

$$(\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_n \tag{15}$$

Drew and Passman [61] introduced the well-known Einstein's formula for evaluating for effective viscosity, μ_{eff},

$$\mu_{eff} = \mu_f(1+2.5\phi) \text{ for } \phi\langle 0.05 \tag{16}$$

Einstein's equation was extended by Brinkman [62] as

$$\mu_{eff} = \mu_f\frac{1}{(1-\phi)^{2.5}} \tag{17a}$$

For $\phi\langle 0.05$, the following expression for μ_{eff} for spherical particles can also be used

$$\mu_{eff} = \mu_f(1+2.5\phi) \tag{17b}$$

Effective thermal conductivity can be incorporated from one of the thermal conductivity model expressions given by Eqs. (1) and (2).

$$K_{eff,Maxwell} = K_1\frac{K_2+2K_1+2(K_2-K_1)\phi}{K_2+2K_1-2(K_2-K_1)\phi} \tag{18}$$

$$K_{eff,Hamilton} = K_1\frac{K_2+(n-1)K_1-(n-1)(K_1-K_2)\phi}{K_2+(n-1)K_1+(K_1-K_2)\phi} \tag{19}$$

where n = shape factor (for sphere $n = 3$, for cylinder $n = 6$) and ϕ = volume fraction of nanoparticles with

$$\mathrm{Pr} = \frac{\mu_{eff}c_{p,eff}}{K_{eff}} \tag{20}$$

$$\mathrm{Re} = \frac{\rho_{eff}ud}{\mu_{eff}} \tag{21}$$

and

$$Nu = \frac{hd}{K_{eff}} \tag{22}$$

For example for fully developed laminar flow under constant wall temperature boundary condition

$$Nu_T = \frac{hd}{K_{eff}} = 3.657 \tag{23}$$

And for the turbulent flow, the Petukhof–Krillov correlation

$$Nu = \frac{hd}{K_{eff}} = \frac{(f/8)\mathrm{RePr}}{1.07+12.7(f/8)^{1/2}(\mathrm{Pr}^{2/3}-1)}\left(\frac{\mu_b}{\mu_w}\right)^n \tag{24}$$

where $n = 0.11$ for $T_w\rangle T_b$, $n = 0.25$ for $T_w\langle T_b$, and $n = 0$ for constant properties of gases with

$$f = (1.82\log_{10}\mathrm{Re} - 1.64)^{-2} \tag{25}$$

can be used to calculate heat transfer coefficients for nanofluids in duct flow for laminar and turbulent flow regimes respectively by the use of the proper nanofluid properties. The most important one is the selection of the suitable thermal and transport properties. The results can be compared with experimental findings under the same conditions.

In the second approach, the governing equations under the specified boundary conditions can be solved. In this case, the equation of conservation (mass, momentum, and energy) which are well known for single-phase flow can be extended for nanofluids. If the micro convective and micro diffusion of the suspended particles (hydrodynamic dispersion) are neglected, these two approaches will result in less heat transfer coefficients than the experimental findings.

Solutions to governing equations can be given assuming the nanofluid is compressible with no slip between the particles and the fluid, but they are in thermal equilibrium. Under such conditions, the general conservation equations in the vectorial form can be written as [63].

Conservation of mass.

$$div(\rho\bar{v}) = 0 \tag{26}$$

Conservation of momentum

$$div(\rho\bar{v}\bar{v}) = -gradP + \mu\nabla^2\bar{v} \tag{27}$$

Conservation of energy

$$div(\rho\bar{v}c_pT) = div(KgradT) \tag{28}$$

These equations can be simplified depending on the required solution.

Because of the several effects, the slip velocity between the ultra-fine nanoparticles and the fluid may not be zero. One must use the dispersed model in which the random movement of the particles is taken into account and the thermal conductivity will be the apparent thermal conductivity. For fully developed nanofluid flow in heated tube, the energy equation in the presence of heat dissipation can be written as

$$U\frac{\partial T}{\partial x} = \left(\alpha_{eff} + \frac{D_x}{(\rho c_p)_{eff}}\right)\frac{\partial^2 T}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}\left[\left(\alpha_{eff} + \frac{D_r}{(\rho c_p)_{eff}}\right) r\frac{\partial T}{\partial r}\right] + \left(\frac{\mu}{\rho c_p}\right)_{eff}\left(\frac{du}{dr}\right)^2 \tag{29}$$

where α_{eff} is the effective diffusivity of nanofluid, D_x and D_r are the thermal dispersion coefficients in axial and radial directions respectively which take into account for the contribution of the hydrodynamic dispersion and the irregular movement of the nanoparticles. If one assumes isotropical flow, that is $D_x = D_r = D$ the effective apparent thermal diffusivity becomes

$$\bar{D} = \alpha^*_{eff} = \alpha_{eff} + \frac{D}{(\rho c_p)_{eff}} \tag{30}$$

and the apparent thermal conductivity

$$\bar{K} = K_{eff} + D \tag{31}$$

For fully developed laminar flow, the velocity profile is parabolic as

$$\frac{u}{\bar{u}} = 2\left(1 - \frac{r^2}{r_0^2}\right) \tag{32}$$

where $\bar{u}$ is the average velocity in the axial direction. If the heat conduction in the axial direction is neglected, then Eq. (29) can be written as for fully developed laminar flow.

$$u\frac{\partial T}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[\bar{D} r\frac{\partial T}{\partial r}\right] \tag{33}$$

The energy equation in any forms must be solved for nanofluid forced convection with the given boundary conditions of constant wall temperature or constant heat flux with a constant inlet temperature.

Researchers in the field of heat transfer enhancement used several techniques for heat transfer enhancement. In recent years, as it is already mentioned, many published articles focused on measuring and determining properties of nanofluids especially modeling and measuring thermal conductivities of nanofluids and heat transfer coefficients, Trisaksri and Wongwises [64].

Xuan and Li [6] presented the classical Graetz solution to Eq. (33) for a plug flow, u = constant, for the constant wall temperature boundary conditions as also given in [63]

$$Nu_x = \frac{h_x d}{K} = \frac{\sum_{m=1}^{\infty} e^{-4\lambda_m^2 (x/d)/Pe}}{\sum_{m=1}^{\infty} \frac{1}{\lambda_m^2} e^{-4\lambda_m^2 (x/d)/Pe}}; \quad Pe = \frac{U_m d}{\alpha^*_{eff}} \tag{34}$$

The classical Graetz problem can be extended for constant heat flux, constant wall temperature, and the linear wall temperature boundary conditions with parabolic velocity profile [63]. Using the parabolic velocity profile, the local Nusselt numbers can be obtained from:

$$Nu_x = \frac{h_x d}{K_{eff}} = \frac{\sum_{n=1}^{\infty} A_n e^{-\lambda_n^2 \xi}}{\sum_{n=1}^{\infty} \frac{A_n}{\lambda_n^2} e^{-\lambda_n^2 \xi}} \text{(constant wall temperature)} \tag{35}$$

$$Nu_x = \frac{h_x d}{K_{eff}} = \left[\frac{11}{48} - \frac{1}{2}\sum_{n=1}^{\infty} \frac{e^{-\beta_n^2 \xi}}{A_n \beta_n^4}\right]^{-1} \text{(constant wall heat flux)} \tag{36}$$

$$Nu_x = \frac{h_x d}{K_{eff}} = \frac{\frac{1}{2} + 4\sum_{n=1}^{\infty} \frac{C_n}{2}\frac{R_n'(1)}{\lambda_n^2} e^{-\lambda_n^2 \xi}}{\frac{88}{768} + 8\sum_{n=1}^{\infty} \frac{C_n}{2}\frac{R_n'(1)}{\lambda_n^4} e^{-\lambda_n^2 \xi}} \text{(linear wall temperature)} \tag{37}$$

where $\xi = \frac{x/r_0}{Pe}$, $Pe = \frac{u_m d}{\alpha_{eff}}$, and eigenvalues and coefficients can be found in Kakaç and Yener [63].

From Eqs. (34)–(37), asymptotic values of Nusselt numbers are obtained. Then using Hamilton and Crosser correlation for the effective thermal conductivity of Al_2O_3/water nanofluid, convective heat transfer coefficient for fully developed laminar flow conditions and the thermal conductivity enhancement are given in Tables 2 and 3. Note that the thermal dispersion coefficient, D_r, is neglected.

As seen from the tables, enhancement in both convective heat transfer coefficient and thermal conductivity increased with increasing particle volume fraction. It should also be noted that the enhancement ratios are the same for convective heat transfer coefficient and thermal conductivity. If the particle volume fraction is increased to 8%, heat transfer coefficient increases 40%.

In the following sections, we further summarize the recent publications on the theoretical investigation of forced convection heat transfer with nanofluids.

Theoretical studies of the possible heat transfer mechanisms have been initiated, but to date obtaining an atomic- and micro-scale-level understanding of how heat is transferred in nanofluids remains the greatest challenge that must be overcome in order to realize the full potential of this new class of heat transfer fluid [65]. They have evaluated four specific mechanisms discussed in the literature could contribute to thermal conductivity which seems to be able to explain the anomalous increases in thermal conductivity enhancement of nanofluids. They discussed the theoretical studies of the possible heat transfer mechanisms of various researchers, and addressed the issue of convective heat transfer enhancement and unresolved problems of heat transfer with nanofluids.

Table 2
Convective heat transfer coefficient and associated enhancement ratios of the fully developed conditions of the base fluid and nanofluid.

Condition		Convective heat transfer coefficient, h, W/m^2 K (percent increase in convective heat transfer coefficient with respect to pure water)		
		0.6% Al_2O_3/water	1.8% Al_2O_3/water	Pure water
1	Slug flow with constant wall temperature boundary condition	177.8 (2.80%)	187.6 (8.51%)	172.9
2	Parabolic velocity profile with constant wall temperature boundary condition	112.4 (2.80%)	118.6 (8.51%)	109.3
3	Parabolic velocity profile with constant wall heat flux boundary condition	134.1 (2.80%)	141.6 (8.51%)	130.5
4	Parabolic velocity profile with linear wall temperature boundary condition	134.1 (2.80%)	141.6 (8.51%)	130.5

Eapen et al. [66] proposed molecular dynamic simulation. He has shown that all the transport coefficients such as thermal conductivity for heat transport and viscosity and diffusion coefficients for momentum and mass transport can be evaluated using the linear response theory and discussed the enhanced phonon transport mechanism in low-dimensional quantum systems such as nanotubes.

Maïga et al. [43] gave a solution of the forced convection of water-γAl_2O_3 and ethylene glycol-γAl_2O_3 nanofluids flowing in a tube under constant and uniform heat flux boundary condition for both laminar and turbulent flow regimes by neglecting the slip velocity between the phases. The governing equations are written in cylindrical coordinate (r, θ, z) and the following expressions have been used to compute the thermal and physical properties of the nanofluids under consideration.

$$\rho_{nf} = (1-\phi)\rho_{bf} + \phi\rho_p \tag{38}$$

$$(c_p)_{nf} = (1-\phi)(c_p)_{bf} + \phi(c_p)_p \tag{39}$$

$$\frac{\mu_{nf}}{\mu_{bf}} = 123\phi^2 + 7.3\phi + 1 \text{ for water} - \gamma Al_2O_3 \tag{40}$$

$$\frac{\mu_{nf}}{\mu_{bf}} = 306\phi^2 - 0.19\phi + 1 \text{ for ethylene glycol} - \gamma Al_2O_3 \tag{41}$$

$$\frac{K_{nf}}{K_{bf}} = 4.97\phi^2 + 2.72\phi + 1 \text{ for water} - \gamma Al_2O_3 \tag{42}$$

$$\frac{K_{nf}}{K_{bf}} = 28.905\phi^2 + 2.8273\phi + 1 \text{ for water} - \gamma Al_2O_3 \tag{43}$$

where p, bf, and nf refer to the particles, the base fluid, and the nanofluid, respectively.

For the turbulent flow, the time averaged Navier–Stokes are solved with k-ε turbulent model. The solution results of the governing equation have been validated by experimental findings. It is also shown that the presence of particles affects on the wall friction which increases with the particle volume concentration.

The augmentation of heat transfer capabilities of radial cooling system was investigated by Roy et al. [44]. They made an analysis of laminar flow and heat transfer in a radial flow of two coaxial and parallel disks cooling system with the use of nanofluids. The governing equations were written in the cylindrical coordinates (r, z) and solved numerically for the hydrodynamic and thermal fields of a water-γAl_2O_3 nanofluid in a radial laminar flow using thermal and physical properties of nanofluids as a classical homogeneous two-phase mixture as used by Maïga et al. [43].

Buongiorno [67] developed a two-component four-equation nonhomogeneous equilibrium model for mass, momentum, and heat transfer in nanofluids. He considered seven slip mechanisms that can produce a relative velocity between the base fluid and nanoparticles. It is conducted that only Brownian diffusion (the random motion of nanoparticles within the base fluid) which results from continuous collision between nanoparticles and the molecules of the base fluid and the thermophoresis (diffusion of particles under the effect of a temperature gradient) are important slip mechanisms in nanofluids and the energy transfer by nanoparticle dispersion is negligible; in the literature, heat transfer enhancement is commonly attributed to nanoparticle dispersion.

Maïga et al. [50] presented a numerical method to study heat transfer enhancement in convective heat transfer by using nanofluids. He investigated the problem of laminar forced convection flow for two particular geometrical configurations of uniformly heated tube and a system of parallel, coaxial, and heated disks, by the use of nanofluids of water-γAl_2O_3 and ethylene glycol-γAl_2O_3. For two boundary conditions considered, correlations have been given for Nusselt number of nanofluids. It has been found that ethylene glycol-γAl_2O_3 yields better heat transfer enhancement than water-γAl_2O_3. But the inclusion of nanoparticles was shown to increase the wall shear stress by as much as 7 times that of the base fluid. It is reasonable to believe this since the nanoparticles also increase the mixture viscosity. Maïga et al. [50] also looked at the effect of particle concentration and flow Reynolds number on the average heat transfer coefficient in the tube using the water based mixture. As a result of the numerical analysis, two correlations for the determination of the averaged Nusselt number for a tube flow as a function of Reynolds number and Prandtl number were provided for the constant heat flux and constant wall temperature boundary conditions. These correlations are:

$$Nu = 0.086\ \mathrm{Re}^{0.55}\mathrm{Pr}^{0.5} \text{ for constant wall heat flux} \tag{44}$$

$$Nu = 0.28\ \mathrm{Re}^{0.35}\mathrm{Pr}^{0.36} \text{ for constant wall temperature} \tag{45}$$

The above correlations are valid for $\mathrm{Re} \leqslant 1000$, $6 \leqslant \mathrm{Pr} \leqslant 753$, and $\phi \leqslant 10\%$.

Koo and Kleinstreuer [48] simulated and analyzed laminar flow in microheat-sinks considering two types of nanofluids, i.e., CuO nanospheres at low consideration in water or ethylene glycol, the conjugated forced convection problem for microheat-sinks. Assuming steady laminar flow of a nanofluid under constant heat flux boundary condition, the governing equations for the fluid and the wall are solved numerically considering the effective thermal conductivity as the same of the conventional static part, K_{static}, as well as a dynamic part, $K_{Brownian}$, which originate from the particle Brownian motion. For the static part, Maxwell model given by Eq. (1) is used and an expression for $K_{Brownian}$ has been developed. For these two mixtures flowing in a microchannel, temperature profiles, and Nusselt numbers are computed. It has been shown that the addition of nanoparticles to high-Prandtl number liquids with nanoparticles at high volume concentration of about 4%, significantly increases the heat transfer performance of microheat-sinks.

Khaled and Vafai [47] presented heat transfer enhancement for fully developed laminar flow in a two-dimensional channel by controlling thermal dispersion effects inside the fluid. In this work energy equations for different fluid regimes are solved with constant heat flux boundary condition analytically and numerically by neglecting axial conduction and heat dissipation. The average properties of nanofluid except the thermal conductivity to account for thermal dispersion effects. The nanofluid is assumed Newtonian. Various distributions for dispersive elements such as nanoparticles are considered and it is shown that the distribution of the dispersive elements that maximizes the heat transfer is governed by the flow and thermal conditions plus the properties of the dispersive elements.

Wang and Mujumdar [68], and Daungthongsuk and Wongwises [69] presented a limited review convective heat transfer using nanofluids which summarizes research work on heat transfer characteristics.

Palm et al. [70] furthered the study of the radial flow cooling systems by including temperature dependent properties; specifically the dynamic viscosity as well as the thermal conductivity were given temperature dependent distributions for nanoparticle concentrations of 1% to 4%. The results then reconfirmed that the use of nanoparticles does increase the heat transfer capabilities for the radial flow cooling system but similar increases were also shown for the wall shear stress. The inclusion of the temperature dependency showed an increase in the heat transfer rate when compared to that of a constant property analysis. However again it is noted that further research must be done to fully understand the behavior and classification of this new cooling fluid.

Maïga et al. [71] presented heat transfer enhancement and studied the hydrodynamic and thermal behavior of turbulent flow

Table 3
Thermal conductivity of nanofluids and pure water and associated enhancement ratios.

	0.6% Al_2O_3/water	1.8% Al_2O_3/water	Pure water
Thermal conductivity, W/m K	0.615	0.649	0.598
Percent increase in thermal conductivity with respect to pure water	2.80%	8.51%	–

in a tube numerically for using Al_2O_3 nanoparticle suspension at various concentrations under the constant heat flux boundary condition. The systems of non-linear and coupled governing equations are solved by a numerical method of control volume. The classical Launder and Spalding [72] k-ε model was employed to model the turbulence. A new correlation as

$$Nu_{fd} = 0.085\mathrm{Re}^{0.71}\mathrm{Pr}^{0.35} \tag{46}$$

is proposed to calculate the fully developed heat transfer coefficient for the water-γAl_2O_3 mixture as a single phase fluid for $10^4 \leqslant \mathrm{Re} \leqslant 5\times10^5$, $6.6 \leqslant \mathrm{Pr} \leqslant 13.9$, and $0 < \phi < 10\%$. Variation of wall shear stress with respect to Reynolds number and particle volume fraction was also examined. Ratio of the wall shear stress of the nanofluid to that of the base fluid was found to be increasing with particle volume fraction. However, it was noted that the ratio was nearly independent of Reynolds number. When absolute value of the wall shear strss was considered it was seen that it increased with both particle volume fraction and Reynolds number.

As mentioned before, convective heat transfer with nanofluids can be modeled using two-phase or single phase approach. The single phase approach assumes that fluid phase and particles are in thermal equilibrium and move with the same velocity. This model has been used in several studies of convective heat transfer with nanofluids [39,43,48,73].

Mirmasoumi and Behzadmehr [74] analyzed laminar mixed convection of a nanofluid consists of water-γAl_2O_3 in a tube numerically by the use of two-phase mixture models. Velocity distributions and Nusselt numbers are obtained over a wide range of Grashof and Reynolds number. By the use of the two-phase model, the effect of the particles movement on the heat transfer coefficient can be clearly understood.

Heris et al. [75] presented a solution for the enhancement of laminar forced convection with a nanofluid flowing in a tube under constant wall temperature boundary condition by the use of the homogeneous model assuming that the flow and energy equations of the base fluid are not affected by the presence of the suspended particles. For the fully developed laminar forced convection, the energy equation for pure fluid with axial and radial conduction was written as

$$u\frac{\partial T}{\partial z} = \left(\frac{K}{\rho c_p}\right)_{nf}\frac{\partial^2 T}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left[\left(\frac{K}{\rho c_p}\right)_{nf} r\frac{\partial T}{\partial r}\right] \tag{47}$$

The parameter $(\rho c_p)_{nf}$ in the above equation for nanoparticles is determined as

$$(\rho c_p)_{nf} = (1-\phi)(\rho c_p)_f + \phi(\rho c_p)_n \tag{48}$$

The property relations given by Eqs. (17b, 48) are introduced in the solution. Yu and Choi correlation given by Eq. (4) is used for the thermal conductivity of nanofluid with $\beta = 0.1$ for determination of nanofluid effective thermal conductivity which is the most important parameter for the enhancement of heat transfer with nanofluids. Dispersion model is also used considering for the contribution of hydrodynamic dispersion and irregular movement of nanoparticles in the radial and axial directions. The energy equation becomes

$$u\frac{\partial T}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left[\left(\frac{K_{nf}}{(\rho c_p)_{nf}} + \frac{K_{d,r}}{(\rho c_p)_{nf}}\right) r\frac{\partial T}{\partial r}\right] + \frac{\partial}{\partial z}\left[\left(\frac{K_{nf}}{(\rho c_p)_{nf}} + \frac{K_{d,z}}{(\rho c_p)_{nf}}\right)\frac{\partial T}{\partial z}\right] \tag{49}$$

The effective thermal conductivity is

$$K_{eff} = K_{nf} + K_d \tag{50}$$

where K_d is the dispersed thermal conductivity and K_{nf} is the thermal conductivity of nanofluid. Neglecting the axial diffusion and dispersion, the energy equation for fully developed laminar flow in a circular tube has been solved at different Peclet numbers (2500–6500) for Al_2O_3–water, Cu–water, an CuO–water nanofluids are validated with experimental findings.

Behzadmehr et al. [76] presented a solution for the prediction of turbulent forced convection of nanofluid in a tube under the constant wall heat flux boundary condition by the use of two-phase approach in which each phase has its own velocity. The nanofluid consisted of water as a base fluid and 1 Vol% Cu. Continuity, momentum and energy equations are written in terms of mean values, volume fraction (ϕ), and the mixture density (ρ_m)

$$\rho_m = \sum_{k=1}^{n}\phi_k\rho_k \tag{51}$$

And μ_m is the mixture viscosity

$$\mu_m = \sum_{k=1}^{n}\phi_k\mu_k \tag{52}$$

where k is the secondary phase, i.e., the nanoparticles. A slip velocity is defined as the velocity of the particle relative to the velocity of the base fluid phase

$$u_{pf} = u_p - u_f \tag{53}$$

The drift velocity is related to the relative velocity

$$u_{dr,p} = u_{pf} - \sum_{k=1}^{n}\frac{\phi_k\rho_k}{\rho_m}u_{f,k} \tag{54}$$

Slip velocity is determined from the following expression [77]

$$u_{pf} = \frac{\rho_p d_p^2(\rho_p - \rho_m)}{18\mu_f f_{drag}\rho_p}a \tag{55}$$

where f_{drag} is given by Schiller and Naumann [78] as

$$f_{drag} = \begin{cases} 1 + 0.15\mathrm{Re}_p^{0.687} & \textit{for}\ \mathrm{Re}_p \leqslant 1000 \\ 0.0183\mathrm{Re}_p & \textit{for}\ \mathrm{Re}_p > 1000 \end{cases} \tag{56}$$

and the acceleration

$$a = g - (u_m\cdot\nabla)u_m \tag{57}$$

The effective solid viscosity is given in terms of solid volume fraction [79]. The turbulence is modeled with the Launder and Spalding [72] k-ε turbulence model for the mixture. In this work, single phase approach is also used and the governing equations are solved by using the same turbulence model. By comparing with experimental results, it is shown that two-phase model is more accurate than the single phase model.

Mirmasoumi and Behzadmehr [80] studied numerically the effect of the nanoparticle size of nanofluid on laminar mixed convection heat transfer in a horizontal tube. Two-phase mixture model based on a single fluid two-phase approach has been used to investigate fully developed mixed convection of water–Al_2O_3 nanofluid. The continuity, momentum, and energy equations for the mixture are employed with uniform heat flux at the wall. The similar method used in this analysis presented by Behzadmehr et al. [76].

5. Conclusions

The literature survey shows that nanofluids significantly improve the heat transfer capability of conventional heat transfer fluids such as oil or water by suspending nanoparticles in these base liquids. Further theoretical modeling and experimental works on the effective thermal conductivity and apparent diffusivity are needed to demonstrate the full potential of nanolfuids for enhancement of forced convection. The understanding of the fundamentals of heat transfer and wall friction is prime importance for developing nanofluids for a wide range of heat transfer application. Although there are recent developments in the study of heat transfer with nanofluids, more experimental results and the theoretical understanding of the mechanisms of the particle movements are needed to understand heat transfer and fluid flow behavior of nanofluids. Further work is also needed for the treatment of nanofluids as a two-phase flow since slip velocity between the particle and base fluid plays important role on the heat transfer performance of nanofluids.

Acknowledgment

The authors thank the Scientific and Technical Research Council (TUBITAK) of Turkey for supporting this study.

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